

MATHEMATICS PRACTICE TEST 3 EXPLANATORY ANSWERS

Question 1. The correct answer is E. If you knew the unknown score, you could check to see that it was correct by adding up all 5 scores, dividing by 5 to get the average, and checking to see that the result was 90. Let x be the unknown score. Then the sum of all of the scores is $85 + 95 + 93 + 80 + x$, and the average is $\frac{85+95+93+80+x}{5}$. For the average to be 90, that means $\frac{85+95+93+80+x}{5} = 90$. To solve, you can multiply both sides by 5 to get $85 + 95 + 93 + 80 + x = 450$, and then subtract 353 from both sides to get $x = 97$.

If you chose B, that is closest to the average of the four given scores, $\frac{85+95+93+80}{4} = 88.25$. To raise an average of 88.25 up to 90 would take a raise of about 2 points, but a single new score of 92 (answer choice D) would not raise the average much. You can check your answer to see that it is too low: $\frac{85+95+93+80+92}{5} = \frac{445}{5} = 89$. You can check any of the answer choices to see whether it is correct.

Question 2. The correct answer is H. When $g = 2$, the value of $g \cdot (g + 1)^2$ is $2 \cdot (2 + 1) = 2 \cdot (3)^2 = 2 \cdot 9 = 18$.

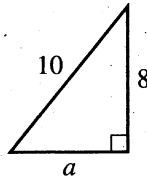
If you chose F, you might have thought of $(g + 1)^2$ as equivalent to the alternate expression $g^2 + 1^2$. But, when $g = 2$, the original has the value $(2 + 1)^2 = (3)^2 = 9$, while the alternate has the value $2^2 + 1^2 = 5$.

Question 3. The correct answer is D. Company A sells at \$15 for 60 pens, which is $\frac{\$15}{60}$ per pen. That reduces to $\frac{\$1}{4}$, or \$0.25 per pen. Company B sells at \$8 for 40 pens, which is $\frac{\$8}{40} = \frac{\$1}{5} = \$0.20$ per pen. Company B is a nickel cheaper.

If you chose A, possibly you found the minimum cost correctly but identified it with the wrong company. Choice C is the average for Company A.

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Question 4. The correct answer is H. The Pythagorean theorem applies here, so that $a^2 + 8^2 = 10^2$, where a is the distance from the base of the ladder to the wall, in feet. That means $a^2 = 10^2 - 8^2 = 36$ and then $a = 6$.



K comes from adding $10^2 + 8^2$, which is 164. If you had noticed that the longest side of the triangle is 10 feet long, and $\sqrt{164}$ is more than 10, you could have eliminated this answer choice. F (2 feet) cannot be true. If it were, the path along the two shorter sides of the triangle (ground and wall) would be the same length as a path along the longest side of the triangle (ladder). This is impossible because the shortest path between points must go along a straight line, not over and then up.

Question 5. The correct answer is E. While you could try out various combinations of the given three statements and try to make a conclusion, it might be more straightforward to look at each of the answer choices to see whether it contradicts one of the given three statements or whether it could be deduced from the given three statements.

A and B each say that Insect I is an ant. This directly contradicts the second given statement. So, A and B are false.

Consider C: if it is true (Insect I is attracted to honey), then the first given statement implies that Insect I is an ant. This contradicts the second given statement. So, C is false.

D directly contradicts the third given statement. So, D is false.

For E, consider Insect J. The third given statement tells you that Insect J is attracted to honey. And then because of the first given statement (all insects attracted to honey are ants), Insect J must be an ant. So, E must be true.

Question 6. The correct answer is F. Each 1,000 gallons of water costs \$2.50, so g of these "1,000 gallons of water" cost $g \cdot (\$2.50)$. On top of this, there is a \$16 charge that is added for trash pickup. The result is $\$2.50g + \16 .

Answer choice K represents a cost of \$2.50 per gallon rather than per 1,000 gallons.

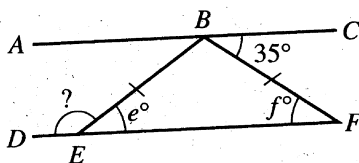
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Question 7. The correct answer is D. The left side of the equation, $2(x + 4)$, can be written as $2x + 8$. Then, the equation becomes $2x + 8 = 5x - 7$. One way to solve this is to first add 7 to both sides (resulting in $2x + 15 = 5x$) and then subtracting $2x$ from both sides (resulting in $15 = 3x$). Dividing both sides by 3 gives the result $5 = x$.

It's a good idea to check this answer: does $2(5 + 4)$ equal $5(5) - 7$? The left side simplifies to $2(9)$, which is 18. The right side simplifies to $25 - 7$, which is also 18. Yes!

Some students will write $2(x + 4)$ as $2x + 4$, but it isn't. That mistake would lead to answer choice C. Note that $2(x + 4)$ is two $(x + 4)$ s added together, or $x + 4 + x + 4$.

Question 8. The correct answer is H. \overline{BF} is a transversal between the two parallel lines. Therefore, because $\angle CBF$ and $\angle BFE$ are alternate interior angles, their measures are equal, and so f° in the figure below is 35° . You can also see that $\triangle BEF$ is isosceles, which makes the base angles equal in measure, so e° is also 35° . The angle the problem asks you to find is the supplement of $\angle BEF$, which makes its measure $180^\circ - 35^\circ$, or 145° .



Question 9. The correct answer is B. The least common denominator is the smallest positive multiple of 2, 3, 9, and 15. These four numbers can be written in prime-power form as 2, 3, 3^2 , and $3 \cdot 5$. The least common denominator must have all of these as factors. The number $2 \cdot 3^2 \cdot 5$ is divisible by 2, by 3, by 3^2 , and by $3 \cdot 5$, and it has no extra prime factors, so it is the smallest of all of the common multiples. $2 \cdot 3^2 \cdot 5 = 2 \cdot 9 \cdot 5 = 10 \cdot 9 = 90$.

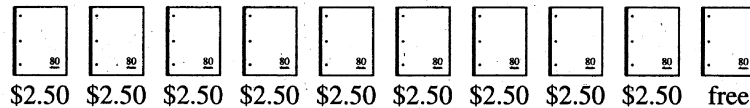
The most popular incorrect answer is A, which is divisible by 15, by 9, and by 3, but not by 2.

Question 10. The correct answer is H. $3x(x^2y + 2xy^2) = 3x \cdot x^2y + 3x \cdot 2xy^2 = 3x^3y + 6x^2y^2$.

F is $3(x^2y + 2xy^2)$, dropping the x multiplier in $3x$. If you chose G, you probably correctly multiplied $3x$ and x^2y but forgot to multiply $3x$ by $2xy^2$.

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Question 11. The correct answer is **D**. The 10 notebooks would cost $9(\$2.50)$, which is \$22.50. The average price for one of the 10 notebooks would be $\frac{\$22.50}{10}$, which is \$2.25.



If you chose **B**, you may not have understood that notebook 10 was free, so you only had to pay for 9 notebooks.

Question 12. The correct answer is **K**. $(3x + 1)^2 = (3x + 1)(3x + 1)$ because that's what the power 2 means. There are several methods for continuing from this point, such as FOIL (first, outer, inner, last). The solution shown below uses the distributive property.

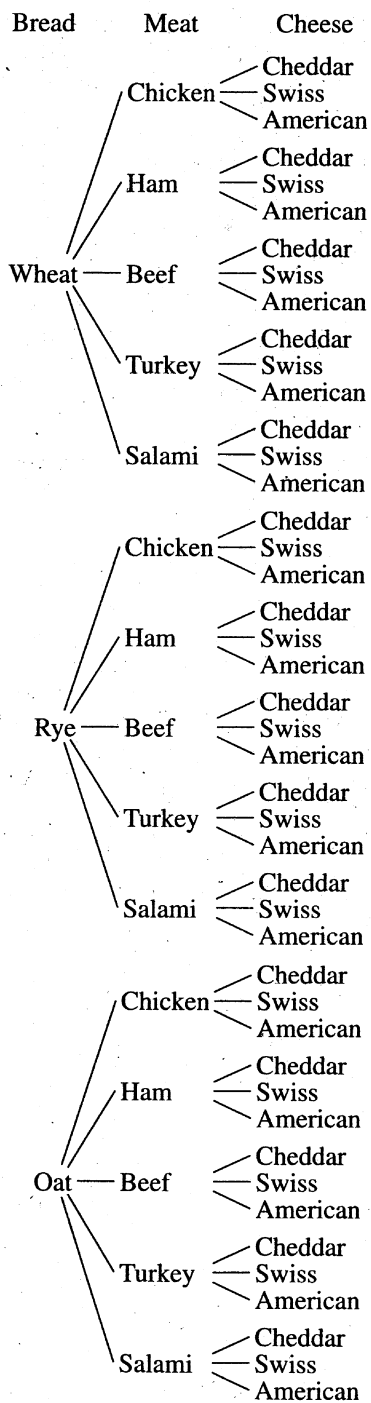
$$(3x + 1)(3x + 1) = 3x(3x + 1) + 1(3x + 1) = 3x(3x) + 3x(1) + 3x + 1 = 9x^2 + 6x + 1$$

It is fairly common for students to think that $(3x + 1)^2$ and $(3x)^2 + (1)^2$ are equivalent, but they're not. If you chose **H**, you may have made this mistake. Understanding why this is a mistake can help you understand other parts of algebra and not make similar mistakes. If you let $x = 1$, then $(3x + 1)^2 = 16$ but $(3x)^2 + (1)^2 = 10$. It makes a difference whether you add first and then square, or whether you square first and then add.

Question 13. The correct answer is **D**. For each kind of bread, there are 5 kinds of meat, so that is $3 \cdot 5$ combinations of bread and meat. For each of these 15 combinations of bread and meat, there are 3 kinds of cheese. That makes $15 \cdot 3 = 45$ combinations of bread, meat, and cheese.

The tree diagram on the next page shows all 45 combinations. It would take a lot of time to list all of these cases, but you can imagine what the tree looks like without having to write it all out. You can see that parts of the tree repeat many times, and so you can use multiplication to help you count.

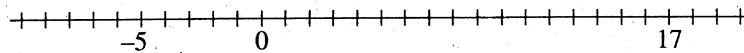
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Question 14. The correct answer is **K**. Because the only real numbers that satisfy $a^2 = 49$ are 7 and -7 , and the only real numbers that satisfy $b^2 = 64$ are 8 and -8 , the only possibilities for $a + b$ are $7 + 8$, $7 + (-8)$, $-7 + 8$, and $-7 + (-8)$. These possibilities are 15, -1 , 1, and -15 . The only answer choice left is 113.

Question 15. The correct answer is **B**. Here's one where drawing a picture might help you avoid some mistakes.



The points with coordinates -5 and 17 are 22 units apart. If you go up by 11 units from -5 , you should get to the same place as if you go down by 11 units from 17 . This is 6.

If you chose **C**, you may have calculated $\frac{17 - (-5)}{2}$. This represents the distance from the midpoint to an endpoint. This expression is very similar to the average of the two coordinates, $\frac{17 + (-5)}{2}$ (notice the plus sign), which is the coordinate of the midpoint. That's why drawing a picture is a good idea.

Question 16. The correct answer is **G**. $3\frac{3}{5} = x + 2\frac{2}{3} \Rightarrow x = 3\frac{3}{5} - 2\frac{2}{3} = (3 - 2) + (\frac{3}{5} - \frac{2}{3}) = 1 + (\frac{9}{15} - \frac{10}{15}) = 1 + (-\frac{1}{15}) = \frac{14}{15}$.

The most common wrong answer is **H**, which happens when someone tries to reduce $(\frac{3}{5} - \frac{2}{3})$ to $(\frac{3-2}{5-3})$.

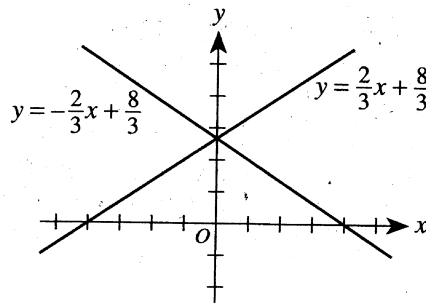
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Question 17. The correct answer is A. Although you could deduce some things the way the equations are given, you might be more comfortable with the equations in slope-intercept form. The system would then be:

$$y = -\frac{2}{3}x + \frac{8}{3}$$

$$y = \frac{2}{3}x + \frac{8}{3}$$

The slopes of the two lines are different (the first is $\frac{2}{3}$, the second $-\frac{2}{3}$). That, by itself, means A is correct. The graph below shows what this system looks like. You can see that none of the other answer choices is correct.



Question 18. The correct answer is K. Properties of exponents make this straightforward to solve. All of the *bases* (2, 4, and 8) are powers of 2. The equation can be rewritten as $(2^x)(2^2) = (2^3)^3$. Properties of exponents lead to the equation $2^{x+2} = 2^{3 \cdot 3}$, which simplifies to $2^{x+2} = 2^9$. If the two exponents are the same, then the left and right sides are equal. This happens when $x + 2 = 9$, which is when $x = 7$.

You can check this answer (or whatever answer you got). When $x = 7$, the left side of the equation is $(2^7)(4)$, which simplifies to $(128)(4)$, and then to 512. The right side is 8^3 , which is also 512.

The most common wrong answer for this problem is G, which happens when people combine $(2^x)(4)$ and get (8^x) . These are not equivalent (check $x = 2$). Another common mistake is to write $(2^3)^3$ as 2^6 , which leads to H.

