NAME Notes DATE _____ PERIOD _____

# 5-7 Study Guide and Intervention

*Roots and Zeros*

## THE RATIONAL ZERO THEOREM

### EXAMPLE 1 List possible rational zeros

List the possible rational zeros of $f$ using the rational zero theorem.

a. $f(x) = x^3 + 2x^2 - 11x + 12$

Factors of the constant term: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$ Answer

List all the factors of the last term (the constant term)

### GUIDED PRACTICE for Example 1

List the possible rational zeros of $f$ using the rational zero theorem.

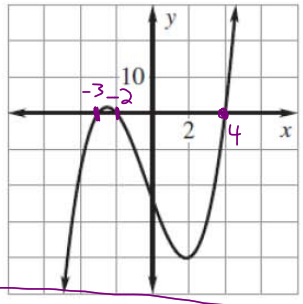
1. $f(x) = x^3 + 9x^2 + 23x + 15$

List
1·15
3·5

Answer = ±1, ±3, ±5, ±15

**Find all rational zeros of the function.**

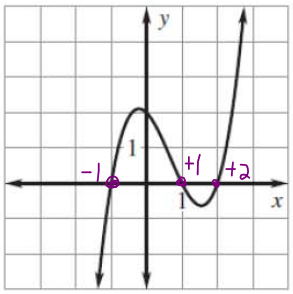
12. $f(x) = x^3 + x^2 - 14x - 24$




List
-1·24
-2·12
-3·8
-4·6
-6 4
-12·2
-24·1

Answers = -3, -2, 4 (from Graph)

13. $f(x) = x^3 - 2x^2 - x + 2$




List
1·2
-1·-2

Answers = -1, +1, +2 (from graph)

## EXAMPLE 2 Find zeros when the leading coefficient is 1

**Find all real zeros of $f(x) = x^3 - 8x^2 + 11x + 20$.** Make a list

**Solution**

***STEP 1*** **List** the possible rational zeros. The leading coefficient is 1 and the constant term is 20. So, the possible rational zeros are:

$$x = \pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}, \pm\frac{20}{1}$$

If x = 1 doesn't work, then try x = - 1

If x = - 1 doesn't work, then try x = 2 ...

***STEP 2*** **Test** these zeros using synthetic division.

**Test $x = 1$:**

| | | | | |
|---|---|---|---|---|
| 1 | 1 | −8 | 11 | 20 |
| | | 1 | −7 | 4 |
| | 1 | −7 | 4 | 24 |

1 is not a zero.

**Test $x = -1$:**

| | | | | |
|---|---|---|---|---|
| −1 | 1 | −8 | 11 | 20 |
| | | −1 | 9 | −20 |
| | 1 | −9 | 20 | 0 |

−1 is a zero.

***STEP 3*** **Factor** the trinomial in $f(x)$ and use the factor theorem.

Because testing x = - 1 worked, we can stop testing.
Then take the numbers from the basement and factor those.

$x^2 - 9x + 20$

List 20

$1 \cdot 20$
$2 \cdot 10$
$4 \cdot 5$
$-4 \cdot -5$

factors $= (x-4)(x-5)$

$\boxed{\text{zeros: } x = 4,\ x = 5,\ x = -1}$

We include x = - 1 in our answer because it gave us a zero when we did synthetic division.