Notes

# 4.6 Perform Operations with Complex Numbers

# DAY 1

## THE SQUARE ROOT OF A NEGATIVE NUMBER

| Property | Example |
|---|---|
| **1.** If $r$ is a positive real number, then $\sqrt{-r} = i\sqrt{r}$. | $\sqrt{-3} = i\sqrt{3}$ |
| **2.** By Property (1), it follows that $(i\sqrt{r})^2 = -r$. | $(i\sqrt{3})^2 = i^2 \cdot 3 = -1 \cdot 3 = -3$ |

## Example 1

**a. Simplify $\sqrt{-48}$.**

$\sqrt{-48} = \sqrt{16 \cdot (-3)}$
$= \sqrt{16} \cdot \sqrt{3} \cdot \sqrt{-1}$
$= 4i\sqrt{3}$

$\sqrt{-1} \cdot \sqrt{48}$
$i \cdot \sqrt{16} \cdot \sqrt{3}$
$i \cdot 4 \cdot \sqrt{3} = 4i\sqrt{3}$

## Example 2

**a. Simplify $-3i \cdot 4i$.**

$-3i \cdot 4i = -12i^2$
$= -12(-1)$
$= 12$

$-3 \cdot i \cdot 4 \cdot i$
$(-3 \cdot 4)(i \cdot i)$
$-12 \cdot (i^2)$
$-12 \cdot (-1) = 12$

## Example 3

| | | | |
|---|---|---|---|
| $i^1 = i$ | $i^2 = -1$ | $i^3 = i^2 \cdot i$ or $-i$ | $i^4 = (i^2)^2$ or 1 |
| $i^5 = i^4 \cdot i$ or $i$ | $i^6 = i^4 \cdot i^2$ or $-1$ | $i^7 = i^4 \cdot i^3$ or $-i$ | $i^8 = (i^2)^4$ or 1 |

$i^{19} = i^{16} \cdot i^3 = i^3 =$ **$-i$**

$i^{52} = i^{48} \cdot i^4 = i^4 =$ **1**

$i^{30} = i^{28} \cdot i^2 = i^2 =$ **$-1$**

$i^{1025} = i^{1024} \cdot i^1 = i^1 =$ **$i$**

## Example 4 *Solve a quadratic equation*

$2x^2 + 15 = -35$

$-15 \quad -15$

$\frac{2x^2}{2} = \frac{-50}{2}$

$\sqrt{x^2} = \sqrt{-25}$

**Original equation**

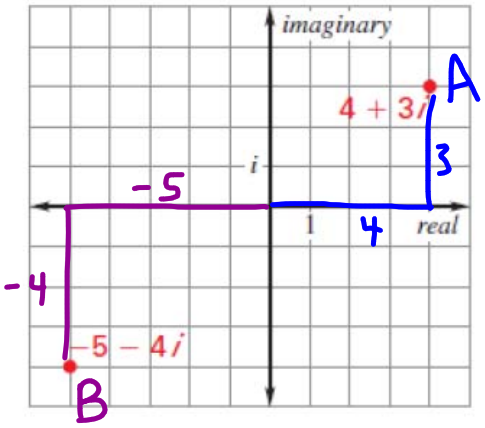
$x = \pm\sqrt{-1} \cdot \sqrt{25}$

$x = \pm i \cdot 5$

$x = \pm 5i$

## Example 5 *Plot complex numbers*

**Plot the complex numbers in the same complex plane.**

**a. $4 + 3i$**

**b. $-5 - 4i$**